Stochastic Multi-stage Hydro Optimization

MAKING BETTER CHOICES ACROSS INFLOW UNCERTAINTY

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Stochastic Hydro Optimization

1.1 INTRODUCTION

Power systems that have both hydro-electric and thermal generation require a systematic and coordinated approach to determine an optimal storage management. The goal of a hydro-thermal planning tool is to minimize the expected thermal costs across the simulation period by dispatching hydro during high-value periods as much as possible. These types of problems generally require stochastic analysis to deal with inflow uncertainty, which increases the mathematical size of the problem and hence are often cumbersome to solve.

This white paper reviews the stochastic analysis algorithm in PLEXOS as applied to different hydro-dominated power systems.

1.2 STOCHASTIC OPTIMIZATION

When operating under uncertainty, a decision-maker must make optimal choices throughout a decision horizon with incomplete information. The decision horizon includes many defined stages, in which each stage represents a point in time where the user either makes a decision, or the uncertainty partially or totally vanishes. The amount of information available to the decision maker is usually different from stage to stage. According to the number of stages in the optimization problem, we can distinguish between:

- Two-stage stochastic problems.
- Multi-stage stochastic problems.

1.2.1 Two-stage Stochastic Problems

Two-stage applies to a decision-making problem in which decisions are made in two stages, and the existing uncertainty is represented by a set of scenarios or samples. It is a two-step process:

a) Make a set of decisions.
b) Profit from the outcome or clean up the mess (recourse actions if any).

The two-stage stochastic optimization doesn't allow the user to make revisions after revealing the uncertainty, so it has the following limitations:

- The plan for the entire horizon is determined before uncertainty is realized.
- Only a limited number of recourse actions (if available) can be taken afterwards.
- Limited use (short-term only) for hydro optimization.

1.2.2 Multi-stage Stochastic Problems

When information is slowly revealed over time, we need to adopt a multi-stage approach. The decision-making process for a multi-stage stochastic problem is:

a) Make a decision for today (***t***) based on what I know today.
b) Observe the outcome from stochastic process in time ***t***.
c) Increment ***t*** and repeat until the entire study horizon has been simulated and optimized.

This type of phased decision-making approach allows users to revise over time as more information regarding the uncertainties is revealed.

Multi-stage stochastic optimization has the following upsides and downsides:

- Upside: It is a better characterization of the dynamic planning process and provides more flexibility than the two-stage model.
- Downside: Simulation size, memory and runtime increase exponentially as we add more stages.

For medium-long term hydro optimization, the most suitable approach is multi-stage stochastic optimization, and that is the type of optimization problem covered in this paper.

Stochastic Tree Reduction for Multi-stage

When using multi-stage stochastic optimization, the user can generate many possible scenarios, as shown in Figure 1. For such a high number of scenarios, it is impossible to numerically obtain a solution for the multi-stage optimization problem. Different techniques have been introduced in the literature to help solve this problem and commonly involve either a scenario tree reduction or a more simplified tree solved with decomposition techniques.

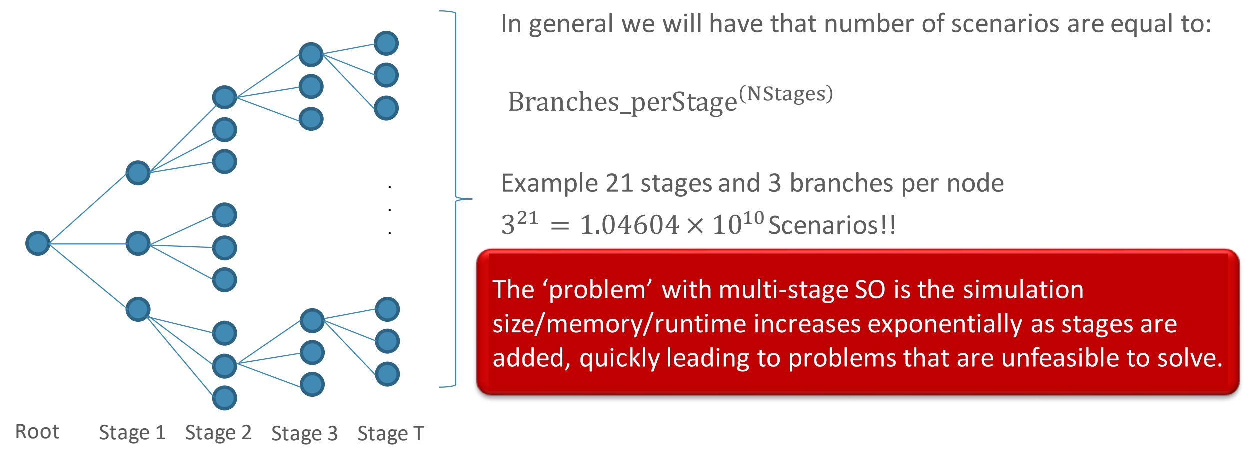

Figure 1: Multi-stage dimensionality issue

PLEXOS offers a scenario-reduction technique called “hanging branches,” which is described in the following section.

2.1 HANGING BRANCHES REPRESENTATION

Assuming we have a multi-stage stochastic problem consisting of four stages and three branches per stage, we can represent the tree as shown in Figure 2.

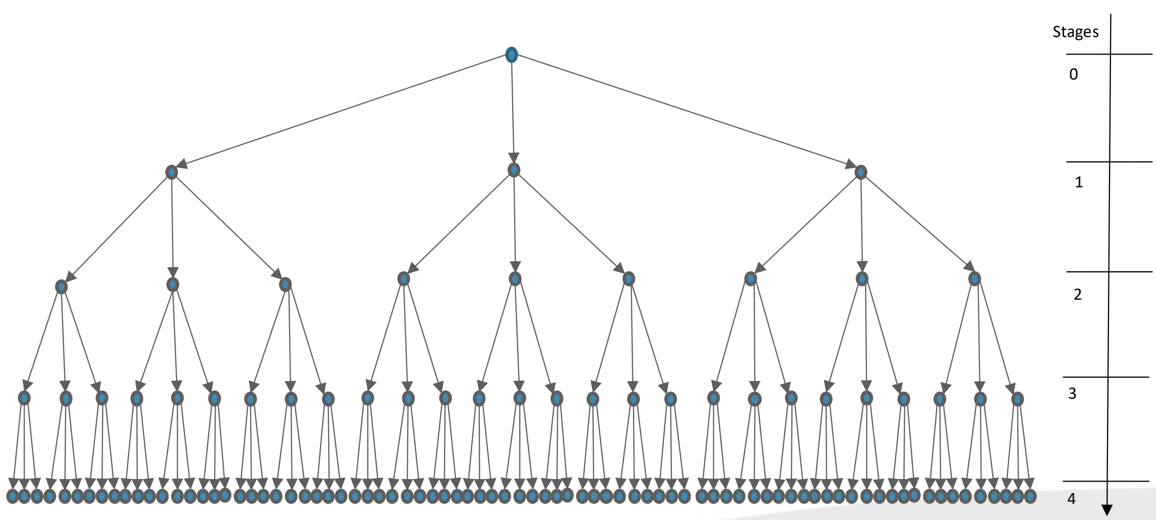

Figure 2: Full multi-stage tree for 4 stages and 3 branches per stage. $3^4 = 81$ *scenarios to explore.*

We can reduce this full tree to solve an equivalent multi-stage problem. There are several ways to reduce the stochastic tree: in this paper we reduce the stochastic tree by organizing the branches as "full branches," "hanging branches" or "death branches," which are defined as:

- Full branch: Path to be explored.
- Hanging branch: Uncertainty associated to the full branch to be explored.
- Death branch: Path to be discarded to avoid dimensionality issues.

If the user reduces the tree in Figure 2, to in three full branches and two hanging branches per stage, Figure 3 is the resulting tree.

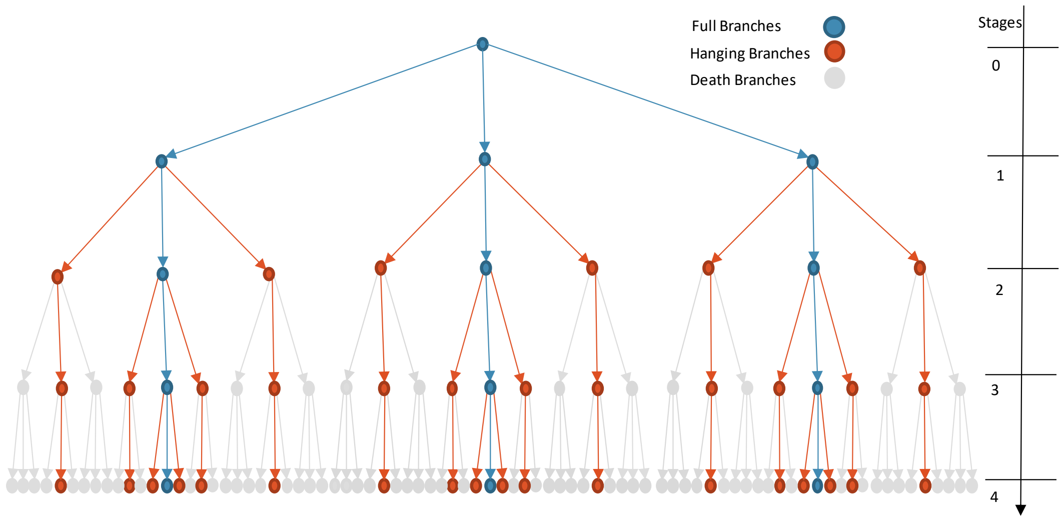

Figure 3: Full tree classified in full, hanging and death branches

By deleting the death branches from the tree in Figure 3, we have the following reduced tree:

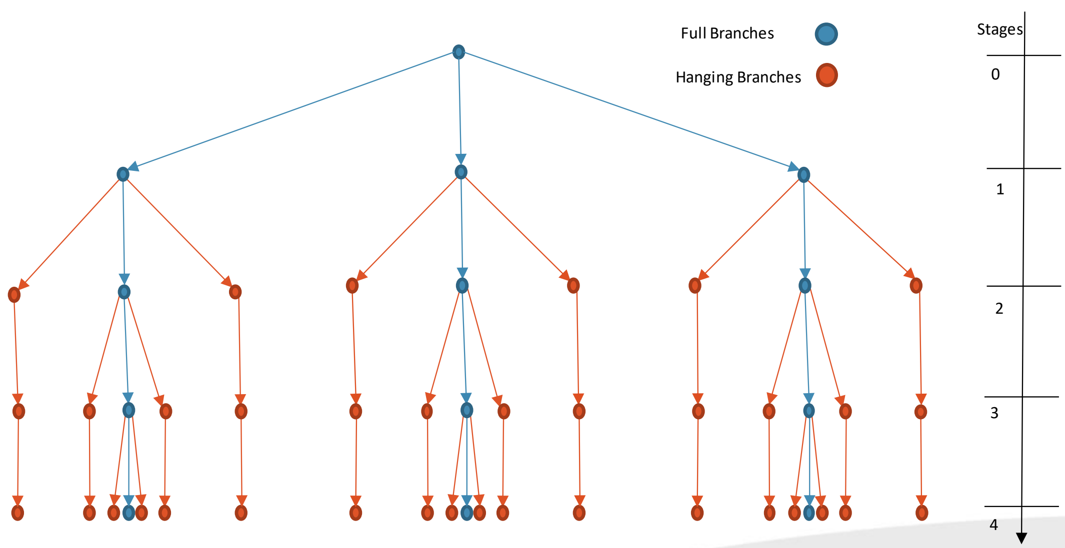

Figure 4: Reduced multi-stage tree.

2.2 INDEPENDENCY

It is important to note that the inflows at the beginning of each stage are known. This means we can solve each full branch independently, as shown in Figure 5.

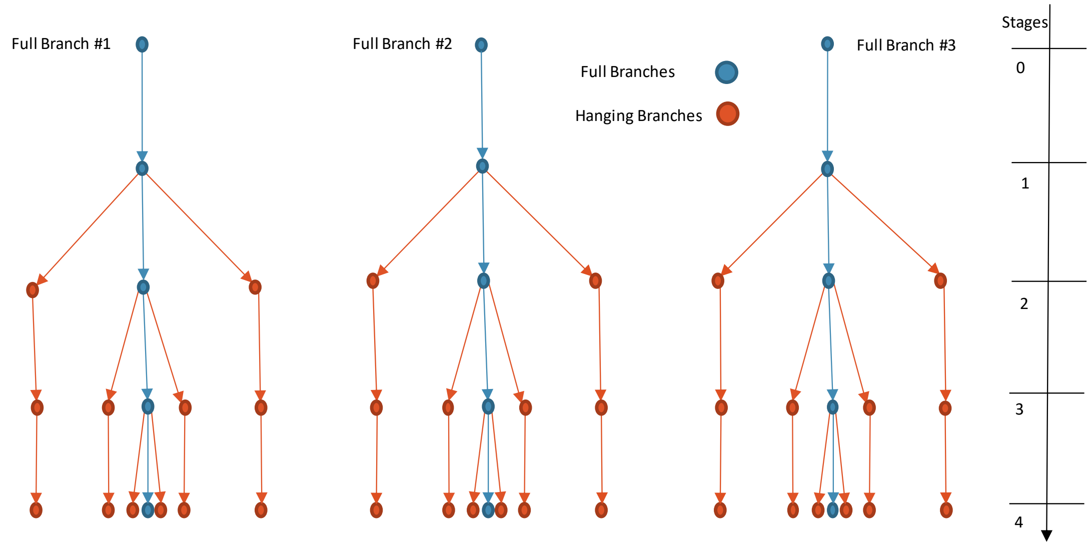

Figure 5: Resulting tree after assuming inflows are known at the beginning of each stage.

2.3 HANGING BRANCHES REDUCTION

By default, the weights for each branch are uniform. In order to avoid dimensionality issues in the number of hanging branches, the user can set the weights used in the objective function for those branches.

We developed a sample reduction algorithm for hanging branches to reduce the number of samples to a predefined smaller number, but the reduced samples are still a good approximation of the original problem. The reduction is based on rules that ensure only the samples that are similar to other samples or have small probabilities are combined. The methodology is illustrated in the following figure:

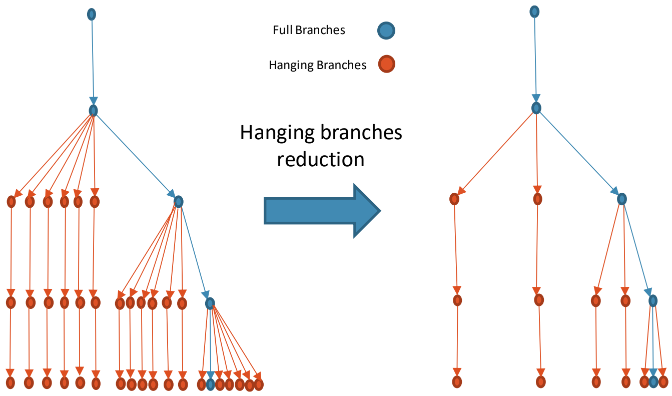

Figure 6: Hanging branches reduction from 6 to 2 hanging branches per stage.

Algorithm to Solve the Problem

To solve the resulting tree shown in Figure 5, PLEXOS 8 offers a methodology called "rolling horizon."

3.1 ROLLING HORIZON

We represent one single, full branch of Figure 5 as follows when solving from a root node:

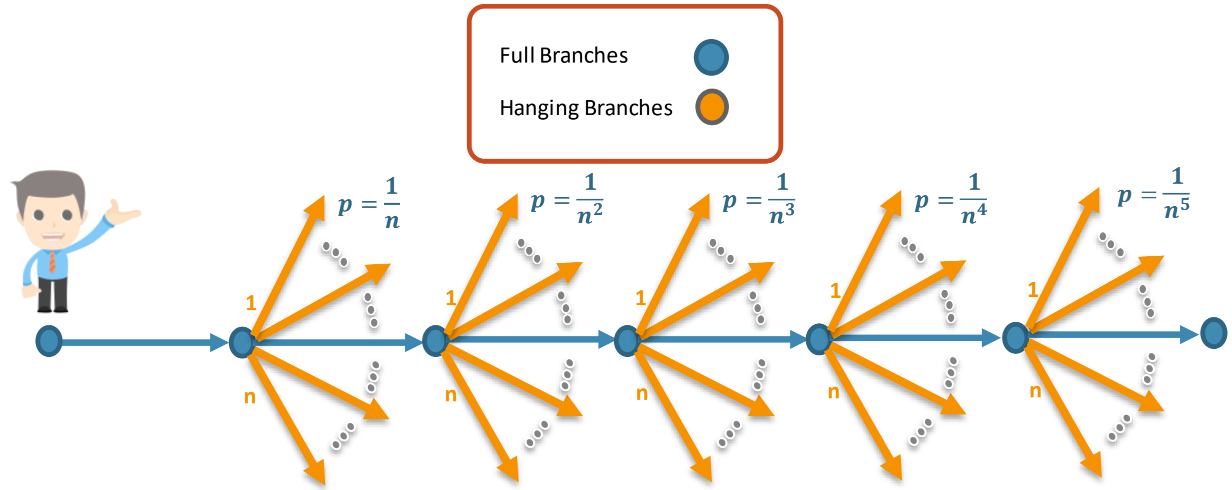

Figure 7: One single full branch with uncertainty at each stage.

If we formulate 20 or 30 hanging branches per stage, the following weights are generated from root node:

Table 1: Weights from root node

Stage	N=20	N=30
1	0.05	0.0333
2	0.0025	0.0011
3	0.000125	$3.7\,x10^{-5}$
4	$6.25x10^{-6}$	$1.23\,x10^{-6}$
5	$3.13x10^{-7}$	$4.12\,x10^{-8}$
6	$1.56x10^{-8}$	$1.37\,x10^{-9}$
7	$7.81x10^{-10}$	$4.57\,x10^{-11}$
8	$3.91x10^{-11}$	$1.52\,x10^{-12}$
9	$1.95x10^{-12}$	$5.08\,x10^{-14}$
10	$9.77x10^{-14}$	$1.69\,x10^{-15}$

Formulating the multi-stage problem in one single optimization problem produces very small weights for future branches. Commercial solvers can't guarantee a solution outside the weight range 10^-6 and 10^6.

However, multi-stage stochastic optimization says that at each stage, the decision-maker can make a new decision because additional information is revealed. We solve this limitation in the same way a multi-stage stochastic problem is solved in real life: by using a "rolling horizon" approach, which splits the horizon in steps. The only required information we must pass between steps is initial storage and ending volumes.

The rolling horizon approach is designed to overcome the limitations of extremely small probabilities deep into the future. The method looks ahead to a given point in the future; the end volumes at that point are passed as initial volumes at the start of the next step.

For a horizon divided in multi-step stochastics, the algorithm works as follows:

Step 1:

a) Starting date is beginning of root node.
b) Multi-stage tree formulated from start to a user-defined stage in the future where no more branches will be added to avoid weights issue.

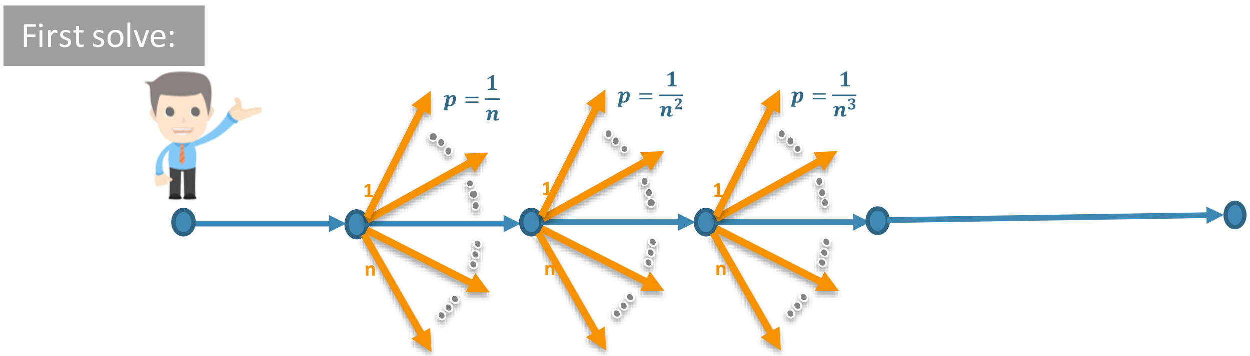

Figure 8: First solve. At some point in future no more branches are formulated to avoid weights issues.

First roll:

a) Starting date is beginning of stage 1.
b) End volumes in first solve are passed as initial volumes in roll 1.
c) The past branches are not formulated because that part of the problem is already solved.
d) More hanging branches are formulated when the weights provide information to the optimization solver.

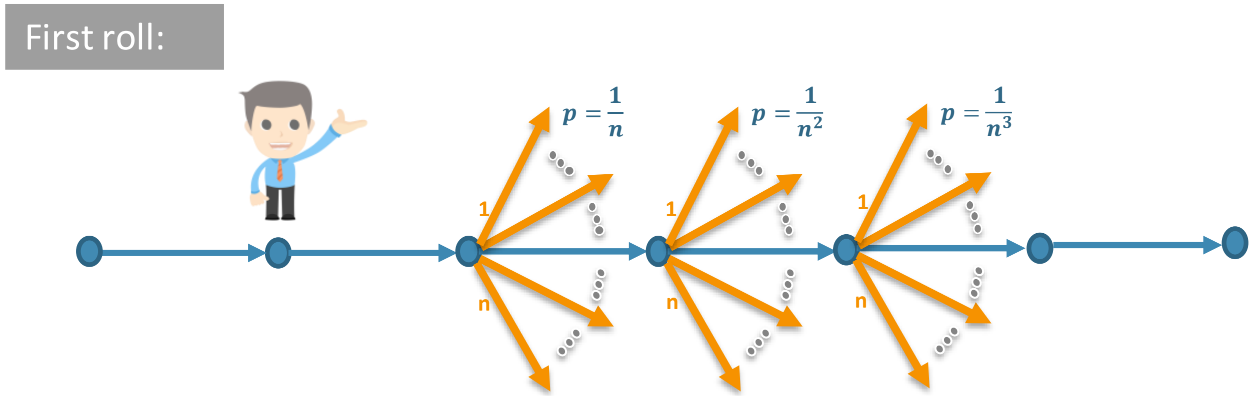

Figure 9: First roll. End volumes of previous step passed as initial volumes. More branches are formulated in the future.

Second roll:

a) Starting date is beginning of stage 2.
b) End volumes in first roll are passed as initial volumes in roll 2.
c) The past branches are not formulated because that part of the problem is already solved.
d) More hanging branches are formulated when the weights provide information to the optimization solver.

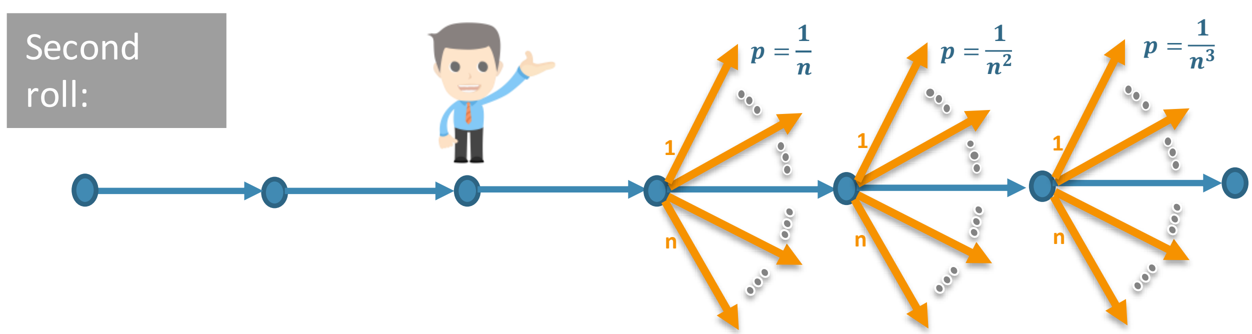

Figure 10: Second roll. End volumes of previous step passed as initial volumes. More branches are formulated in the future.

The problem continues rolling until it has fully explored the horizon.

Synthetic Sample Generation

Multi-stage stochastic optimization needs a large number of scenarios, or samples, to represent the equivalent tree. The literature includes different methods for creating these samples, however the simplest method creates future samples based on history.

If we have historical records from 2000 to 2011, and the user wants to create samples from 2019 to 2030, the we need to generate the number of full branches as described in the following table:

Table 2: Historical information

	Simulated Year							
Full Branches		2019	2020	2021	…	2028	2029	2030
	1	2000	2001	2002	…	2009	2010	2011
	2	2001	2002	2003	…	2010	2011	2000
	3	2002	2003	2004	…	2011	2000	2001
	…	…	…	…	…	…	…	…
	…	…	…	…	…	…	…	…
	…	…	…	…	…	…	…	…
	12	2011	2000	2001	…	2008	2009	2010
	…	…	…	…	…	…	…	…

This method is powerful and simple, as it preserves the spatial correlation between hydro storages and doesn't require statistical analysis.

To generate the hanging branches, we can use the historical information as well. For this purpose, the sampling method selects a different year for the same position in time.

A weekly multi-stage problem starting from historical year 2000 could look as follows:

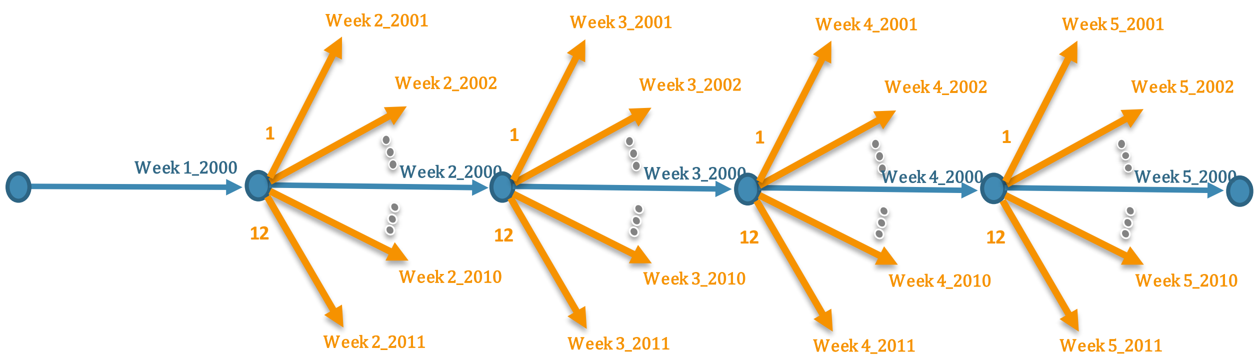

Figure 11: 1 full branch. Weekly stages.

The hanging branches for full branch number 1 looks as follows:

Table 3: Hanging branches associated to full branch # 1

	Simulated Stage							
Hanging Branches		Stage 1	Stage 2	Stage 3	…	Stage 10	Stage 11	…
	1	Week 2_2001	Week 3_2001	Week 4_2001	…	Week 11_2001	Week 12_2001	…
	2	Week 2_2002	Week 3_2002	Week 4_2002	…	Week 11_2002	Week 12_2002	…
	3	Week 2_2003	Week 3_2003	Week 4_2003	…	Week 11_2003	Week 12_2003	…
	…	…	…	...	…	...	…	...
	…	…	…	…	…	…	…	…
	…	…	…	…	...	…	...	…
	12	Week 2_2012	Week 3_2012	Week 4_2012	…	Week 11_2012	Week 12_2012	…
	…	…	…	…	…	…	…	…

Numerical Examples

We analyzed the following real cases with rolling horizon and hanging branches in PLEXOS 8 R03 using a machine with the following specifications:

Type	Value
Physical Memory	379.87
CPU Type	Intel® Xeon® Platinum 8175M CPU @ 2.50 GHz
CPU/Core Count	48

5.1 CHILE TWO YEARS OPERATIONAL PLANNING

Table 4: Power System Size for Chilean Case

Objects	Number of objects
Storages	11
Thermal generators	251
Hydro Generators	154
Renewable Generators	129
Nodes	29
Lines	79

Table 5: Simulation Settings Chilean Case

Setting	Value
Horizon	2 years
Number of stages	104
Number of blocks per stage	5
Stage ends at each	Week
Full branches	55
Hanging branches	55 reduced to 10

5.1.1 Performance Results

Table 6: Performance for Chilean Case

Step	Run Time		
Root 0	00:01:35	Roll 11	00:01:15
Roll 1	00:01:24	Roll 12	00:01:31
Roll 2	00:01:38	Roll 13	00:01:16
Roll 3	00:01:38	Roll 14	00:01:17
Roll 4	00:01:16	Roll 15	00:01:13
Roll 5	00:01:22	Roll 16	00:01:08
Roll 6	00:01:18	Roll 17	00:01:26
Roll 7	00:01:15	Roll 18	00:01:18
Roll 8	00:01:33	Roll 19	00:01:23
Roll 9	00:01:21	Roll 20	00:00:37
Roll 10	00:01:51		

Total Run Time (including writing results): 37 min 53 secs

5.2 BRAZIL SEVEN YEARS OPERATIONAL PLANNING

Table 7: Power System Size for Brazil Case

Objects	Number of objects
Storages	229
Thermal generators	140
Hydro Generators	229
Renewable Generators	38
Nodes	1
Lines	-

Table 8: Simulation Settings Brazil Case

Setting	Value
Horizon	7 years
Number of stages	84
Number of blocks per stage	3
Stage ends at each	Month
Full branches	84
Hanging branches	10

5.2.1 Performance Results

Table 9: Performance for Brazil Case

Step	Run Time		
Root 0	3:00	Roll 9	4:50
Roll 1	4:49	Roll 10	5:18
Roll 2	5:27	Roll 11	4:20
Roll 3	5:33	Roll 12	4:53
Roll 4	4:51	Roll 13	4:17
Roll 5	4:43	Roll 14	4:34
Roll 6	4:35	Roll 15	4:41
Roll 7	5:23	Roll 16	2:11
Roll 8	4:32		

Total Run Time (including writing results): 1 hour 17 min 20 secs

5.3 NEW ZEALAND 2 YEARS OPERATIONAL PLANNING

Table 10: Power System Size for New Zealand Case

Objects	Number of objects
Storages	83 (9 controlled)
Thermal generators	142
Hydro Generators	39
Renewable Generators	36
Nodes	22
Lines	52

Table 11: Simulation Settings New Zealand Case

Setting	Value
Horizon	104 Weeks
Number of stages	104
Number of blocks per stage	15 (no reduction for HB)
Stage ends at each	Week
Full branches	85
Hanging branches	85 reduced to 10

5.3.1 Performance Results

Table 12: Performance for New Zealand Case

Step	Value		
Root 0	00:04:43.6	Roll 11	00:06:11.0
Roll 1	00:05:52.8	Roll 12	00:05:21.3
Roll 2	00:06:12.5	Roll 13	00:04:46.9
Roll 3	00:06:31.9	Roll 14	00:04:26.4
Roll 4	00:05:57.0	Roll 15	00:03:55.7
Roll 5	00:05:37.2	Roll 16	00:03:29.1
Roll 6	00:06:35.9	Roll 17	00:03:00.8
Roll 7	00:07:26.6	Roll 18	00:02:33.8
Roll 8	00:06:34.5	Roll 19	00:02:02.2
Roll 9	00:06:37.6	Roll 20	00:00:53.5
Roll 10	00:06:47.8		

Total Run Time (including writing results): 1 hours 56 min 35 secs

5.4 NORDIC SYSTEM 2 YEARS OPERATIONAL PLANNING

Table 13: Power System Size for Nordic Case

Objects	Number of objects
Storages	8
Thermal generators	722
Hydro Generators	995
Renewable Generators	508
Nodes	17
Lines	51

Table 14: Simulation Settings Nordic Case

Setting	Value
Horizon	104 Weeks
Number of stages	104
Number of blocks per stage	12 (HB=1)
Stage ends at each	Week
Full branches	12
Hanging branches	10

5.4.1 Performance Results

Table 15: Performance for Nordic Case

Step	Run Time		
Root 0	00:02:22.5	Roll 11	00:02:28.9
Roll 1	00:02:15.8	Roll 12	00:03:08.0
Roll 2	00:01:35.3	Roll 13	00:03:06.2
Roll 3	00:01:36.7	Roll 14	00:02:43.3
Roll 4	00:02:24.4	Roll 15	00:02:35.9
Roll 5	00:02:52.0	Roll 16	00:02:38.9
Roll 6	00:02:26.9	Roll 17	00:02:56.9
Roll 7	00:02:19.7	Roll 18	00:03:11.7
Roll 8	00:01:57.3	Roll 19	00:03:13.7
Roll 9	00:01:52.1	Roll 20	00:01:48.6
Roll 10	00:02:12.3		

Total Run Time (including writing results): 55 min 20 secs

Conclusions

We tested "rolling horizon hanging branches" methodology in real case examples. Performance is affected by the number of full and hanging branches, as well as the level of transmission detail in the model. The simulation times we obtained are comparable to the current methodologies available to solve the multi-stage stochastic hydro problem.

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